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Quasiconformal maps in metric spaces with controlled geometry

by

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1. Introduction

This paper develops the foundations of the theory of quasiconformal maps in metric spaces that satisfy certain bounds on their mass and geometry. The principal message is that such a theory is both relevant and viable.

The first main issue is the problem of definition, which we next describe. Quasiconformal maps are commonly understood as homeomorphisms that distort the shape of infinitesimal balls by a uniformly bounded amount. This requirement makes sense in every metric space. Given a homeomorphism f from a metric space X to a metric space Y, then for $x\in X$ and $r>0$ set

$$H_f(x,r)=\frac{\sup\{|f(x)-f(y)|:|x-y|\leqslant r\}}{\inf\{|f(x)-f(y)|:|x-y|\geqslant r\}}. \tag{1.1}$$

Here and hereafter we use the distance notation $|x-y|$ in any metric space.

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