

# Two cardinal invariants of the continuum ( $\mathfrak{d} < \mathfrak{a}$ ) and FS linearly ordered iterated forcing

by

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## Contents

§0. *Introduction.*

§1.  $\text{CON}(\mathfrak{a} > \mathfrak{d})$ . We prove the consistency of the inequality mentioned in the title, relying on the theory of CS-iteration of nep forcing (from [S8], this proof is a concise version).

§2. *On  $\text{CON}(\mathfrak{a} > \mathfrak{d})$  revisited with FS, non-transitive memory of non-well-ordered length.* This does not depend on §1. We define “FSI-template”, a depth on the subsets on which we shall do induction; we are interested just in the cases where the depth is some ordinal (and not  $\infty$ ). Now the iteration is defined and its properties are proved simultaneously by induction on the depth. After we have understood such iterations sufficiently well, we proceed to prove the consistency in details.

§3. *Eliminating the measurable.* In §2, for checking the criterion which appears there for having “ $\mathfrak{a}$  large”, we have used ultrapower by some  $\kappa$ -complete ultrafilter. Here we construct templates of cardinality, e.g.  $\aleph_3$ , which satisfy the criterion; by constructing them such that any sequence of  $\omega$ -tuples of appropriate length has a (big) subsequence which is “convergent”, so some complete  $\kappa$ -complete filter behaves for an appropriate  $\kappa$ -sequence of names of reals as if it is an ultrafilter and as if the sequence has appropriate limit.

§4. *On related cardinal invariants.* We prove, e.g., the consistency of  $\mathfrak{u} < \mathfrak{a}$ . Here the forcing notions are not so definable, so this gives a third proof of the main theorem (but the points which repeat §3 are not repeated).