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On discrete Möbius groups in all dimensions:
A generalization of Jørgensen's inequality

by

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Introduction

In this paper we present a generalization to all dimensions of the following sharp inequality of Jørgensen [Jø] concerning discrete nonelementary groups:

THEOREM (Jørgensen's inequality). *Let f and g be Möbius transformations of the Riemann sphere. If f and g together generate a discrete nonelementary group, then*

$$|\mathrm{tr}^2(f)-4|+|\mathrm{tr}[f,g]-2|\geqslant 1.$$

Here $[f,g]=fgf^{-1}g^{-1}$ is the multiplicative commutator and we are identifying f and g with their matrix representatives in $\mathcal{SL}_2\mathbf{C}$. Nonelementary in this setting means not virtually Abelian.

It is the following principle that makes Jørgensen's inequality such a valuable tool.

If two Möbius transformations f and g generate a nonelementary discrete group, then given f, g cannot be too close to the identity.

In higher dimensions the trace seems not to be such a good invariant and moreover we must work in the Lie group $\mathcal{O}^+(1,n)$. Here is the generalization we propose:

THEOREM. *Let f and g be Möbius transformations of $\mathbf{S}^n$. If f and g together generate a discrete nonelementary group, then*

$$\max\{\|g^ifg^{-i}-\mathrm{Id}\|: i=0,1,2,..,n\}\geqslant 2-\sqrt{3}.$$

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