Acta Math., 178 (1997), 1–37

General existence theorems for Hamilton–Jacobi equations in the scalar and vectorial cases

by

BERNARD DACOROGNA and PAOLO MARCELLINI

École Polytechnique Fédérale de Lausanne
Lausanne, Switzerland

Università di Firenze
Firenze, Italy

Contents

1. Introduction
2. The quasiconvex case
3. The nonconvex scalar case and systems of equations
4. The convex case (scalar and vectorial)
5. The prescribed singular values case
6. Appendix: Some approximation lemmas
7. Appendix: Polyconvexity, quasiconvexity, rank-one convexity
References

1. Introduction

We consider the Dirichlet problem for *Hamilton–Jacobi equations* both in the scalar and in the vectorial cases. We deal with the following problem:

$$\begin{cases} F(Du(x))=0, & \text{a.e. } x\in\Omega, \\ u(x)=\varphi(x), & x\in\partial\Omega, \end{cases} \tag{1.1}$$

where Ω is a (bounded) open set of $\mathbf{R}^n$, $F:\mathbf{R}^{m\times n}\to\mathbf{R}$ and $\varphi\in W^{1,\infty}(\Omega;\mathbf{R}^m)$. We emphasize that $u:\Omega\subset\mathbf{R}^n\to\mathbf{R}^m$, with $m,n\geqslant 1$, is a *vector valued function* if $m>1$ (otherwise, if $m=1$, we say that u is a *scalar function*). As usual Du denotes the gradient of u.

This problem (1.1) has been intensively studied, essentially in the scalar case in many relevant articles such as Lax [28], Douglis [23], Kružkov [27], Crandall–Lions [16], Crandall–Evans–Lions [14], Capuzzo Dolcetta–Evans [8], Capuzzo Dolcetta–Lions [9], Crandall–Ishii–Lions [15]. For a more complete bibliography we refer to the main recent monographs of Benton [7], Lions [29], Fleming–Soner [25], Barles [6] and Bardi–Capuzzo Dolcetta [5].