THE DIOPHANTINE EQUATION $ax^3+by^3+cz^3=0$. COMPLETION OF THE TABLES

BY

ERNST S. SELMER

of Oslo

§ 1. In a previous paper [2], I have studied the cubic curve

$$X^3 + Y^3 = A Z^3, \tag{1}$$

giving the number of generators and the basic rational solutions for nearly all positive (cube-free) integers $A \leqq 500$. The solutions were found by means of a "first descent", leading to equations of the form

$$ax^3 + by^3 + cz^3 = 0, \quad abc = A \tag{2}$$

$$3auv(u-v) + b(u^3 - 3u^2v + v^3) = 3A_1w^3, \quad A_1(a^2 - ab + b^2) = A, \tag{3}$$

and by a "second descent" in certain cubic fields defined by these equations.

The extensive tables of [2][1] contain a few blank spaces, where no solution had been found, but where my congruence conditions of the second descent did not show insolubility. In some of these cases, the corresponding equations can be proved insoluble by the methods of Cassels [1], showing *the insufficiency of my conditions* (§§ 2–3 below).

The remaining unsolved equations of [2] have all been solved on the electronic computer at the Institute for Advanced Study in Princeton, N. J. (§ 4; the completion of Tables 2^{a-b}, 5 and 6). Consequently, I now have *the complete solution of* (1) *for all* $A \leqq 500$.

One of my earlier conjectures concerning the equation (2) is incorrect and must be modified (§ 5; Tables 2^{c-d}).

---

[1] There is a misprint in the last line of Table 3, for $p=17$: *for* $w=0$ *read* $w=1$.