ON NORMALIZABLE TRANSFORMATIONS IN HILBERT SPACE.

By

H. J. ZIMMERBERG

of New Brunswick, New Jersey.

1. **Introduction.** Zaanen [4] has recently extended the theory of normal transformations in Hilbert space, developed by Rellich [2], to normalizable transformations, and has applied his results to certain special systems of Fredholm integral equations

$$(1) \qquad \lambda \psi_i(x) = \sum_{j=1}^{n} \int_{\Delta} K_{ij}(x, y)\, \psi_j(y)\, dy \qquad (i = 1, \ldots, n).$$

Now, if the adjoint of a normalizable transformation can be defined in the space (as is always possible in a complete Hilbert space), then we shall show that the existence Theorem 10 of Zaanen [4] can be extended. If, in addition, the normalizable transformation is completely continuous (as is the case for the kernels of (1) above), then a further extension will be obtained, and in this case one of the hypotheses of Theorem 12 of [4] may be omitted. This result will then be applied in § 3 to integral systems (1) which are definitely self-conjugate adjoint or J-definite according to [5], which is a generalization of the definite systems treated by Reid [1] and Wilkins [3].

The notation of Zaanen [4] will be employed. Spaces will be denoted by capital German letters and transformations by capital Roman letters. If the adjoint transformation of K exists it will be denoted by K^*.

2. **Existence theorems.** Let $\mathfrak{R}$ be a Hilbert space with elements $f, g, h, \phi, \ldots$ and inner product (f, g). Let H denote a bounded, positive, self-adjoint transformation on $\mathfrak{R}$ to $\mathfrak{R}$; i.e., a bounded linear transformation satisfying $(Hf, g) = (f, Hg)$ and $(Hf, f) \geq 0$ for arbitrary f and g of $\mathfrak{R}$. The set of all elements h for which $Hh \equiv 0$ shall be designated by $[\mathfrak{L}]$, while $[\mathfrak{M}]$ shall denote the orthogonal complement of $[\mathfrak{L}]$. We shall assume that every $f \in \mathfrak{R}$ is expressible