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Uniformization of Kähler manifolds with vanishing Bochner tensor

by

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Dedicated to Professor Frank Raymond for his sixtieth birthday

Introduction

In 1948, S. Bochner introduced a curvature tensor on Hermitian manifolds [1]. He defined it as an analogue to the Weyl conformal curvature tensor. When, on a Riemannian manifold M^n, the Weyl conformal curvature tensor ($n>3$) or the Schouten–Weyl tensor ($n=3$) vanishes, then M^n is said to be a conformally flat manifold. In this case, M^n can be uniformized over the n-sphere S^n with respect to the group of conformal transformations $\mathrm{Conf}(S^n)$. It is natural in Geometry to determine the class of compact Kähler manifolds for which the Bochner curvature tensor vanishes. The Bochner curvature tensor B on a complex manifold with a Kähler metric is defined as follows:

$$\begin{aligned} B_{\alpha\bar\beta\varrho\bar\sigma} = R_{\alpha\bar\beta\varrho\bar\sigma} - \frac{1}{n+2}(R_{\alpha\bar\beta}g_{\varrho\bar\sigma} + R_{\varrho\bar\beta}g_{\alpha\bar\sigma} + g_{\alpha\bar\beta}R_{\varrho\bar\sigma} + g_{\varrho\bar\beta}R_{\alpha\bar\sigma}) \\ + \frac{R}{(n+1)(n+2)}(g_{\alpha\bar\beta}g_{\varrho\bar\sigma} + g_{\varrho\bar\beta}g_{\alpha\bar\sigma}). \end{aligned}$$

Here, $R_{\alpha\bar\beta\varrho\bar\sigma}$ is the curvature tensor, and $R_{\varrho\bar\sigma} = R_{\alpha}{}^{\alpha}{}_{\varrho\bar\sigma}$ and $R = g^{\varrho\bar\sigma}R_{\varrho\bar\sigma}$ are the Ricci tensor and the scalar curvature respectively.

The purpose of this note is to show that when the Bochner curvature tensor B vanishes with respect to a Kähler metric g, the Kähler manifold M^{2n} can be uniformized over the Kähler manifold $Y^n_{\mathbf{C}}$ with a canonical Kähler metric with respect to the transitive group $\mathcal{G}$ consisting of transformations preserving the geometric structure of $Y^n_{\mathbf{C}}$.

Recall that a uniformization of M^{2n} is a maximal collection of charts $\{(\phi_\alpha, U_\alpha)\}_{\alpha\in\Lambda}$ satisfying the following: