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Extrapolation of Carleson measures and the analyticity of Kato's square-root operators

by

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1. Introduction, history and statement of the main theorem

Let A be an $(n\times n)$-matrix of complex L^∞-coefficients, defined on $\mathbf{R}^n$, with $\|A\|_\infty\leqslant\Lambda$, and satisfying the ellipticity (or "accretivity") condition

$$\lambda|\xi^2|\leqslant \operatorname{Re}\langle A\xi,\xi\rangle\leqslant\Lambda|\xi|^2, \tag{1.1}$$

for $\xi\in\mathbf{C}^n$ and for some λ,Λ such that $0<\lambda\leqslant\Lambda<\infty$. Here $\langle\cdot,\cdot\rangle$ denotes the usual inner product in $\mathbf{C}^n$, so that

$$\langle A\xi,\xi\rangle\equiv\sum_{i,j}A_{ij}(x)\xi_j\cdot\bar{\xi}_i.$$

We define a divergence-form operator

$$Lu\equiv-\operatorname{div}(A(x)\nabla u), \tag{1.2}$$

which we interpret in the usual weak sense via a sesquilinear form.

The accretivity condition (1.1) enables one to define an accretive square root $\sqrt{L}\equiv L^{1/2}$ (see [14]), and a fundamental question is to determine when one can solve the "square-root problem", i.e. to establish the estimate

$$\|\sqrt{L}f\|_{L^2(\mathbf{R}^n)}\leqslant C\|\nabla f\|_{L^2(\mathbf{R}^n)}, \tag{1.3}$$

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