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A global calculus of parameter-dependent pseudodifferential boundary problems in L_p Sobolev spaces

by

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Introduction

The theory of pseudodifferential boundary problems has been developed to provide a larger framework for the study of differential boundary value problems, allowing algebraic manipulations with the operators (reflected in a symbolic calculus) and allowing the inclusion of non-local terms. The elliptic calculus has its origin in works of Vishik, Èskin and Boutet de Monvel (cf. [V-E], [E], [BM1] and in particular the Acta article [BM2]) and was further developed e.g. in Rempel and Schulze [R-S1] and Grubb [G1]. The scope of the theory was enlarged by the consideration of systems *depending on a parameter* (running in a noncompact set), which can be for example a spectral parameter $\lambda\in\mathbf{C}$ (allowing functional calculus), a time dependence (for parabolic problems) or a small parameter $\varepsilon>0$ (entering in singular perturbation problems). For operators in L_2 spaces, such a theory was worked out in the book [G2], and further developed for parabolic problems by Grubb and Solonnikov [G-S1], who applied it to give new results on fully nonhomogeneous Navier–Stokes problems (cf. [G-S2] and its references). Let us also mention the treatment in [R-S2] of resolvent estimates and complex powers for systems without the so-called transmission property.

The purpose of the present work is to extend the parameter-dependent calculus to the L_p setting, $1<p<\infty$, and to a suitable class of unbounded manifolds, including exterior domains (complements of smooth compact sets) in $\mathbf{R}^n$ and $\overline{\mathbf{R}}^n_+$.

A fundamental difficulty in the study of parameter-dependent elliptic pseudodifferential problems, depending e.g. on a spectral parameter λ on a ray in $\mathbf{C}$, is the following: Without the parameter, the singularity of the homogeneous symbols at $\xi=0$ is harmless (since it is felt in a compact set only), but when λ is adjoined, the singularity has an