MEAN VALUES OVER THE SPACE OF LATTICES

BY

C. A. ROGERS

in Birmingham

1. Various methods have been used for calculating the mean value of a function, defined for all lattices of determinant 1, over some or all the lattices of determinant 1. It is accepted that the most natural way of calculating such an average is in terms of the invariant measure used by Siegel.[1] However, the averaging methods used by Rogers[2] and by Cassels[3] are more convenient to use, while the methods used by Mahler,[4] by Davenport and Rogers[5] and by Macbeath and Rogers[6] are more appropriate for the special problems considered. The method used recently by Rogers[7] has proved to be particularly convenient. The first object of this paper is to establish a close connection between this averaging method and Siegel's method, but while we will confine our attention to the relationship between this particular averaging method and Siegel's method, it will be clear from the nature of our proofs that the averaging methods mentioned above, except those used by Mahler, and by Davenport and Rogers, will stand in a similar relationship to Siegel's method.

Let $\Lambda = \Lambda(\theta_1, \theta_2, \ldots, \theta_{n-1}, \omega)$ denote the lattice generated by the points

$$\begin{aligned}
\boldsymbol{A}_1 &= (\omega, 0, \ldots, 0, \theta_1 \omega^{-n+1}),\\
\boldsymbol{A}_2 &= (0, \omega, \ldots, 0, \theta_2 \omega^{-n+1}),\\
&\cdots\cdots\cdots\cdots\\
\boldsymbol{A}_{n-1} &= (0, 0, \ldots, \omega, \theta_{n-1} \omega^{-n+1}),\\
\boldsymbol{A}_n &= (0, 0, \ldots, 0, \omega^{-n+1}).
\end{aligned}$$

[1] C. L. Siegel, *Annals of Math.*, 46 (1945), 340–347. We assume that the reader is familiar at least with pages 341 and 342 of Siegel's paper.

[2] C. A. Rogers, *Annals of Math.*, 48 (1947), 994–1002, and a paper to be published in the *Phil. Trans. Royal Soc.* (1955).

[3] J. W. S. Cassels, *Proc. Cambridge Phil. Soc.*, 49 (1953), 165–166.

[4] K. Mahler, *Duke Math. Jour.*, 13 (1946), 611–621.

[5] H. Davenport and C. A. Rogers, *Duke Math. Jour.*, 14 (1947), 367–375.

[6] A. M. Macbeath and C. A. Rogers, to appear in the *Proc. Cambridge Phil. Soc.*

[7] *loc. cit.* (1955).

17 – 553810. *Acta Mathematica*. 94. Imprimé le 15 décembre 1955.