ON THE MAXIMUM TERM AND THE RANK OF AN ENTIRE FUNCTION

BY

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1. Let $f(z)=\sum_{n=0}^{\infty} a_n z^n$ be an entire function and let $\mu(r)=\mu(r, f)$ be the maximum term of the series for $|z|=r$ and $\nu(r)$ the rank of the maximum term. Let R_n be the points of discontinuity of $\nu(r)$. Let $\mu'(r)$ and $\nu'(r)$ correspond to $f'(z)$ and in general $\mu^k(r)$ and $\nu^k(r)$ correspond to $f^k(z)$.

In section 1 we prove results concerning the maximum term $\mu(z)$ and in sections 2, 3, and 4, results concerning $\mu(r)$ and $\nu(r)$.

THEOREM 1. If $f(z)$ be an entire function of order $\varrho<1$, then

$$\frac{r^p \mu^k(r)}{\mu(r)} \to 0 \text{ as } r\to\infty$$

for

$$k=1, 2, 3, \ldots$$

and

$$p<k(1-\varrho).$$

REMARK. If $p=k(1-\varrho)$, then the result is not necessarily true. Take $k=1$ and consider $f(z)=\cos\sqrt{z}$.

$$\varrho(f)=\tfrac{1}{2}.$$

For

$$(2n-1)\,2n\le r<(2n+1)(2n+2)$$

$$\mu(r, f)=\frac{r^n}{(2n)!}\sim\frac{r^n}{(2n)^{2n+\frac{1}{2}}e^{-2n}\sqrt{2\pi}}\sim\frac{e^{\sqrt{r}}}{\sqrt{2\pi}\,r^{\frac{1}{4}}}.$$

Similarly, we can show that

$$\mu(r, f')\sim\frac{1}{2\sqrt{2\pi}}\frac{e^{\sqrt{r}}}{r^{\frac{3}{4}}}.$$

Hence

$$\frac{\mu(r, f')\,r^{\frac{1}{2}}}{\mu(r, f)}\sim\tfrac{1}{2}.$$