# The density of integer points on homogeneous varieties

by

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## A. The setting

### 1. Introduction

Let $V$ be a homogeneous algebraic set in $\mathbf{C}^s$ defined over the rationals, i.e. a set

$$V = V(\underset{\sim}{\mathfrak{F}}) = V(\mathfrak{F}_1, \ldots, \mathfrak{F}_r),$$

consisting of the common zeros of given forms $\mathfrak{F}_1, \ldots, \mathfrak{F}_r$ of positive degrees, in $s$ variables, and with rational coefficients. We are interested in

$$z_P(V) = z_P(\underset{\sim}{\mathfrak{F}}),$$

the number of integer points $\underset{\sim}{x} = (x_1, \ldots, x_s)$ on $V$ with

$$|\underset{\sim}{x}| := \max(|x_1|, \ldots, |x_s|) \leq P.$$

Not much is known in general about the behaviour of $z_P(V)$ as a function of $P$. In those cases where we do have information and where $z_P(V) \to \infty$ (i.e. where $V$ contains an integer point besides $\underset{\sim}{0}$) we have

$$z_P(V) \sim \mu P^\beta,$$

where $\mu > 0$, $\beta > 0$ and $\beta$ is an integer.

Birch [1] could show that a system $\underset{\sim}{\mathfrak{F}}$ of $r$ forms of odd degrees $\leq k$ in $s > c_1(k, r)$ variables possesses a nontrivial integer zero. In particular, $z_P(\underset{\sim}{\mathfrak{F}}) \to \infty$. It would be easy

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