

The inverse spectral problem for self-adjoint Hankel operators

by

A. V. MEGRETSKII⁽¹⁾

V. V. PELLER⁽²⁾

and

S. R. TREIL⁽³⁾

*University of Newcastle
Newcastle, NSW, Australia*

*University of Hawaii
Honolulu, HI, U.S.A.*

*Michigan State University
East Lansing, MI, U.S.A.*

Contents

Introduction	242
I. Necessary conditions	248
I.1. Spectral multiplicities of self-adjoint Hankel operators . . .	249
I.2. Eigenvalues of Hankel operators	253
II. Balanced realizations with continuous time	255
II.1. Hankel operator and balanced realizations	256
II.2. Unitary equivalence modulo the kernel	257
II.3. Lyapunov equations	260
II.4. The main construction	262
II.5. The kernel of Γ_h	268
II.6. Exercises in measure theory	275
II.7. Positive Hankel operators with multiple spectrum	277
III. Balanced realizations with discrete time	279
III.1. Hankel operators and linear systems	280
III.2. Output normal realizations of Hankel operators	281
III.3. Balanced realizations	284
III.4. Asymptotic stability	288
III.5. The main construction	289
III.6. The point spectrum of A^* on the unit circle	294
III.7. How to cope with Q	295
III.8. The Fredholm spectrum of $\mathcal{J}\mathcal{J}$	297
III.9. Compact integral operators	300
III.10. A theorem in perturbation theory	306
References	308

⁽¹⁾ Also affiliated to St. Petersburg University, St. Petersburg, Russia.

⁽²⁾ Also affiliated to the St. Petersburg Branch of the Steklov Mathematical Institute, St. Petersburg, Russia. Research supported by an NSF grant in modern analysis.

⁽³⁾ Also affiliated to St. Petersburg University, St. Petersburg, Russia. Research supported by an NSF grant in modern analysis.