THE NUMBERS OF LABELED COLORED AND CHROMATIC TREES

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1. Introduction

The mathematical theory of trees, as first discussed by Cayley in 1857 [3], was concerned in the enumeration aspect with two contrasting cases; the points (or lines) of the trees in question were either all alike or all unlike. For the allied subject of series-parallel electrical networks, R. M. Foster [5] has introduced enumerations by two variables, the number of elements in the network (corresponding to lines of a tree) and the number of these which are marked, with each mark distinct from every other. Two networks which differ only by a permutation of marks are counted as different if the differently marked elements are dissimilar, with dissimilarity as explained in [5] and [2]. The same kind of enumeration is done here for trees, thus including both classical cases in one frame. The trees so marked are called labeled trees.

A second kind of marking is also considered. This is familiar from graph coloring, where each point or line of a graph is given one of c colors. Note that every element (point or line) of a colored tree has some color (if uncolored elements were permitted, they could all be said to be colored with a new color $c+1$) and that in any particular coloring, any number of different colors (up to c) may appear, in contrast to a labeled tree. The enumeration is by number of elements (points or lines) and by number of colors, or by number of elements and by number of distinct colors.

It may be noted that the enumerating functions for colored trees are formally similar to those for unmarked trees, while both differ from those for labeled trees. This formal similarity does not persist when tree colorings are subject to the chromatic condition that adjacent elements (points having a line in common or lines having