THE TRIANGULATION OF LOCALLY TRIANGULABLE SPACES[1]

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We shall be concerned with the following triangulation problem: "May a space which can be triangulated locally be triangulated in the large?" A locally triangulable space is a separable metric space every point of which has a neighborhood which is homeomorphic to an open subset of some finite polyhedron. Special cases of this problem include the triangulation problems for manifolds, for manifolds with boundary, and for differentiable manifolds; these problems have been attacked in the past, with varying degrees of success. In 1935, G. Nöbeling published an argument for the triangulation theorem for manifolds [7], but it contained an essential error [9]. Subsequently, S. S. Cairns proved triangulability for differentiable manifolds [3], [4], and T. Radó triangulated the general manifold of dimension two [8]. More recently, E. E. Moise proved that three-dimensional manifolds are triangulable [5], and both Moise and R. H. Bing extended this to three-dimensional manifolds with boundary [6], [2]. In the present paper, the author uses some of these results to prove triangulability for locally triangulable spaces of dimension three or less.

We attack the general triangulation problem by attempting to reduce it to the triangulation problem for n-manifolds with boundary. This approach proves successful for n not greater than three. The proofs, while complicated, are elementary in the sense that they involve no algebraic topology.

In Chapter I certain basic definitions and lemmas are given. A new definition of locally polyhedral space is introduced in Chapter II, and some of the implications of this definition are studied. In Chapter III a space X^*, called the composition space of X, is defined; it is in some respects the opposite of a decomposition space. The technical device of passing from a space to its composition space enables us, in Chapter IV, to treat the general triangulation problem.

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