On the vanishing of and spanning sets for Poincaré series for cusp forms

by

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§ 0. Introduction and summary of results

0.1. Let Γ be a finitely generated non-elementary Kleinian group with region of discontinuity $\Omega=\Omega(\Gamma)$ and limit set $\Lambda=\Lambda(\Gamma)$. Let $\lambda(z)\,|dz|$ be the Poincaré metric on Ω (normalized to have constant negative curvature -1). Fix $q\in\mathbf{Z}$, $q\geq 2$. A *cusp form for* Γ *of weight* $(-2q)$ is a holomorphic function φ on Ω that satisfies

$$\varphi(\gamma z)\,\gamma'(z)^q=\varphi(z),\quad \text{all } z\in\Omega,\ \text{all } \gamma\in\Gamma, \tag{0.1.1}$$

and

$$\iint_{\Omega/\Gamma}\lambda(z)^{2-q}|\varphi(z)\,dz\wedge d\bar{z}|<\infty. \tag{0.1.2}$$

Condition (0.1.2) is equivalent to

$$\sup\{\lambda(z)^{-q}|\varphi(z)|;\ z\in\Omega\}<\infty. \tag{0.1.3}$$

Denote by $\mathbf{A}_q(\Omega,\Gamma)$ the space of cusp forms for Γ of weight $(-2q)$.

If $\infty\in\Omega$, then for $\varphi\in\mathbf{A}_q(\Omega,\Gamma)$,

$$\varphi(z)=O(|z|^{-2q}),\quad z\to\infty.$$

We will be studying spaces of rational functions. A rational function f will be considered to be *holomorphic at* ∞ if

$$f(z)=O(|z|^{-2q}),\quad z\to\infty.$$

We will consider it to have a *simple pole at* ∞ if

([1]) Research partially supported by NSF grant MCS 8102621.