CONTRIBUTIONS TO HARMONIC ANALYSIS

In Memory of the School of Analysis of H. Hahn, E. Helly, J. Radon, at the University of Vienna

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1. The problem

Wiener's approximation theorem was the starting point of many developments in harmonic analysis. Carleman, in his proof of the theorem [1], introduced a new method which is of considerable generality and leads to the formulation and solution of new approximation problems. These problems are of the following type:

In the space[2] $L^1(G)$ of integrable functions on a locally compact abelian group G a closed linear subspace I is given which is invariant under "translations", i.e., which contains with a function $f_0(x)$ also all functions $f_0(ax)$, for arbitrary $a \in G$.

It is required to find, for a given function $f(x)$ in $L^1(G)$, the number

$$\inf_{f_0 \in I} \int |f(x) - f_0(x)|\, dx$$

which indicates how closely $f(x)$ may be approximated, in the metric of $L^1(G)$, by means of the functions belonging to I. Using geometrical language, this number is called the *distance* of $f(x)$ from the linear subspace I and denoted by dist $\{f, I\}$.

As is well known, this distance is the norm in the quotient-space $L^1(G)/I$ (cf. [3], Theorem 22.11.4; since $L^1(G)$ is a (commutative) Banach algebra, with convolution as multiplication, and I an ideal in $L^1(G)$, $L^1(G)/I$ is actually a quotient-algebra). The exact calculation of the distance makes it possible to determine explicitly the structure of $L^1(G)/I$.

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[2] Notation and terminology are as usual; cf., e.g., [4]. In particular, dx denotes the Haar measure, and integration extends over the whole group G unless otherwise specified.