A BEURLING-TYPE THEOREM

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§ 1. Introduction and statement of the main result

In this paper we shall be concerned primarily with the linear topological space $A^{-\infty}$ whose elements are holomorphic functions

$$f(z)=\sum_0^\infty a_\nu z^\nu$$

in the unit disk $U=\{z\colon |z|<1\}$ satisfying

$$|f(z)|\leqslant C_f(1-|z|)^{-n_f}\quad (z\in U) \tag{1.1}$$

or equivalently,

$$\log^+|a_\nu|=O(\log \nu)\quad (\nu\to\infty).$$

$A^{-\infty}$ can be thought of as the union of Banach spaces A^{-n} $(n>0)$, the norm in each A^{-n} being defined as follows:

$$\|f\|_{-n}=\sup_{z\in U}\{|f(z)|(1-|z|)^n\}<\infty. \tag{1.2}$$

The topology in $A^{-\infty}$ is introduced in a standard way [6]. Clearly, $A^{-\infty}$ is a topological algebra under pointwise multiplication. It is the smallest algebra containing the disk algebra A (1) and closed under differentiation.

The dual of $A^{-\infty}$ is the topological algebra A^{∞} whose elements are functions $F(z)$ holomorphic in U and infinitely differentiable in $\bar{U}$:

$$F(z)=\sum_0^\infty b_\nu z^\nu\quad (b=O(\nu^{-k})\quad \forall k>0). \tag{1.3}$$

The linear functionals in $A^{-\infty}$ are given by the formula

$$F(f)=\frac{1}{2\pi i}\lim_{r\to 1-}\int_{\partial U}\bar{F}(\zeta)f(r\zeta)\frac{d\zeta}{\zeta}=\sum_0^\infty \bar{b}_\nu a_\nu. \tag{1.4}$$

(1) A is the algebra of all functions continuous in $\bar{U}$ and analytic in U with sup-norm and pointwise multiplication.

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