

# RESIDUAL INTERSECTIONS AND TODDS FORMULA FOR THE DOUBLE LOCUS OF A MORPHISM

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## § 1. Introduction

The main purpose of the following article is to present a proof of a formula of J. A. Todd for the rational equivalence class of the double locus of a morphism. In our proof, the main ingredient is a formula for the rational equivalence classes of residual intersections without embedded components which is of considerable interest in itself. The latter formula we shall derive from an important special case called the “formule clef”, conjectured by Grothendieck ([3], exposé 0, 1957) and proved by A. T. Lascu, D. Mumford and D. B. Scott [8]. In his article [10], Todd obtained his formula for the double locus and a formula closely related to our formula for residual intersections simultaneously, by an inductive argument. It is interesting to notice that while we can prove the latter formula under mild conditions on the intersections involved, the formula for the double locus is shown to be true only under restrictive transversality conditions on the morphisms in question. We shall however, give a weaker version of Todd’s formula for the double locus, which follows immediately from the formula for residual intersections and which holds under correspondingly mild transversality conditions.

In a previous article [7] we treated the particular case of Todd’s formula for the double locus when the target variety was a projective space. We could in that case, define the scheme of double points as the scheme of zeroes of a section of a locally free sheaf. As a consequence, when the section intersected the zero section properly, we obtained an easy proof of the weak version of Todd’s formula, avoiding the “formule clef”. We also proved that when the morphism in question was induced by a generic projection (possibly after a twisting of the embedding involved) the morphism satisfied the restrictive conditions under which Todd’s formula holds. As a consequence of our results we obtained generalizations of results of A. Holme [5] and of C. A. M. Peters and J. Simonis [9] about secants of projective schemes. These results are generalized further below.