CANONICAL POLYGONS FOR FINITELY GENERATED FUCHSIAN GROUPS

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1. Introduction

We consider finitely generated Fuchsian groups G. For such groups Fricke defined a class of fundamental polygons which he called canonical. The two most important distinguishing properties of these polygons are that they are strictly convex and have the smallest possible number of sides. A canonical polygon P in Fricke's sense depends on a choice of a certain "standard" system of generators S for G. Fricke proved that for a given G and S canonical polygons always exist. His proof is rather complicated. Also, Fricke's polygons are not canonical in the technical sense; there are infinitely many P for a given G and S.

In this paper, we shall construct a *uniquely determined* fundamental polygon P which satisfies all of Fricke's conditions for every given G of positive genus and for every given S. We call this P a canonical Fricke polygon.

For every given G of genus zero, and S, we shall define a *uniquely determined* fundamental polygon P which we call a canonical polygon without accidental vertices. From this P, one can obtain in infinitely many ways polygons satisfying Fricke's conditions.

Our canonical polygons are invariant under similarity transformations of the group G if G is of the first kind. If G is of the second kind, this statement remains true after a suitable modification which will be clear from the construction.

The proof involves elementary explicit constructions, and continuity arguments which use quasiconformal mappings, as developed by Ahlfors and Bers.

We give a geometric interpretation of the canonical polygons in the last section. This interpretation provided the heuristic idea for the formulation of the main theorem.