Transformation groups on complex Stiefel manifolds

by

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0. Introduction

In the theory of transformation groups a most fundamental, but in general quite difficult problem, is the classification of the possible orbit structures for actions of a compact Lie group G on a given space X. The well known P. A. Smith theory (as generalized by Borel, Conner, and others) gives beautiful results when X is of the simplest topological type (e.g. acyclic, cohomology sphere, cohomology projective space) and G is a torus or a p-torus. Moreover, when G is a classical group, restriction of the action to the maximal torus of G combined with structural splitting theorems on the characteristic class level for torus actions, result in nice regularity theorems for classical group actions on spaces of such simple topological type ([H1]).

It is our assertion that the time is ripe for applying more sophisticated methods now available in algebraic topology and equivariant cohomology theory in a more serious study of transformation groups on certain spaces of more complicated topological types. The most natural spaces to consider are various homogeneous spaces, which accomodate a rich variety of natural actions. In this paper we give the full proof for one starting theorem in the field of large transformation groups on homogeneous spaces. Our main result is:

THEOREM 1. *Let $X=W_{n,k}$ be the complex Stiefel manifold of $(n-k)$-frames in complex n-space $\mathbf{C}^n$, $k>n/2$, and let $G=SU(n)$. Then any non-trivial, smooth action of G on X is conjugate to the linear action.*

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