New Banach space properties of the disc algebra and H^∞

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0. Introduction

The purpose of this paper is to prove some new linear properties of the disc algebra A and the space H^∞ of bounded analytic functions on the disc. More precisely, results on absolutely summing operators, cotype, finite rank projections and certain sequence properties, such as Dunford-Pettis property and weakly completeness, are obtained.

The main motivation for this work were A. Pelczynski's notes (see [44]), which contain also most of the required prerequisites. Our work extends [44], since it solves several of the main problems. It is also of interest in connection with questions raised in [30], [32], [33], [35], [59]. Besides [44], our references for Banach space theory are [36], [37], [38], [47]. Basic facts about H^p-spaces can be found in [18], [20], [27], [53], [54].

In what follows, we will first describe the frame of the work and recall some definitions. Then we will summarize the several sections of the paper and state the main results. If u is an operator from a space X into a space Y and $0<p<\infty$, we say that u is p-absolutely summing provided there is a constant λ such that

$$\sum \|u(x_i)\|^p \leq \lambda^p \max\left\{\sum |\langle x_i, x^*\rangle|^p;\quad x^*\in X^*,\ \|x^*\|\leq 1\right\}$$

holds for all finite sequences (x_i) of elements of X. The p-summing norm $\pi_p(u)$ of u is the smallest λ with above property. Let $\Pi_p(X, Y)$ be the space of p-summing operators from X into Y.

For $0<p<1$, the spaces $\Pi_p(X, Y)$ coincide and will also be denoted by $\Pi_0(X, Y)$, the 0-summing operators from X into Y. Say that u is p-integral, resp. strictly p-integral, provided u admits a factorization

$$\begin{tikzcd} X \arrow[r, "u"] \arrow[d, "S"'] & Y \arrow[r, "j"] & Y^{**} \\ L^\infty(\mu) \arrow[r, "I"] & L^p(\mu) \arrow[ur, "T"'] & \end{tikzcd} \qquad \text{resp.} \qquad \begin{tikzcd} X \arrow[r, "u"] \arrow[d, "S"'] & Y \\ L^\infty(\mu) \arrow[r, "I"] & L^p(\mu) \arrow[u, "T"'] \end{tikzcd}$$

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