

# SUBALGEBRAS OF $C^*$ -ALGEBRAS II

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## Introduction

This paper continues the study of non self-adjoint operator algebras on Hilbert space which began in [1]. Chapter 1 concerns dilation theory. The main results (1.2.2 and its corollary) imply that every commuting  $n$ -tuple of operators having a general compact set  $X \subseteq \mathbb{C}^n$  as a “complete” spectral set has a (commuting) normal dilation whose joint spectrum is contained in  $\partial X$ , the Silov boundary of  $X$  relative to the rational functions which are continuous on  $X$ . This is a direct generalization of a known dilation theorem for single operators having for a spectral set a compact set  $X \subseteq \mathbb{C}$  with connected complement, and it seems to clarify the relation between spectral sets and normal dilations. In section 1.3 we discuss non-normal dilations and present a result along these lines.

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