FACTORISABLE REPRESENTATIONS OF CURRENT GROUPS AND THE ARAKI-WOODS IMBEDDING THEOREM

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1. Introduction

In order to study the current commutation relations of quantum field theory, Araki and Woods [2] and Araki [1] introduced the notion of current groups and factorisable representations of such groups. Araki and Woods [2] and Streater [5] established that such representations admit a natural imbedding in a symmetric Fock space $\exp H$ over a Hilbert space H. If G is a locally compact group, then a suitable space of Borel functions on a Borel space with values in G is made into a group under pointwise multiplication. This is called a current group of G. Araki [1] established that the factorisable representations of the current group are based on certain cocycle valued measures. In this paper we show the existence of a measure on the Borel space over which the current group is constructed, relative to which a cocycle valued density exists. This yields a certain natural topology for the current group under which the factorisable representation is continuous.

In order to take into account all the cocycles of first order in the construction of factorisable representations it turns out that projective representations should also be considered. Finally, the Araki-Woods imbedding is explicitly constructed in terms of the cocycles. At this stage it may be worth remarking that our methods differ very much from that of Araki and Woods. We rely more on measure theory and not at all on lattice theory.

2. Araki functions

Throughout this paper H with or without suffixes will always stand for a complex separable Hilbert space with inner product $\langle \cdot, \cdot \rangle$. Let G be a fixed locally compact second countable group with identity element e. By a representation of G in H we shall always mean a continuous homomorphism of G into the group $\mathcal{U}(H)$ of unitary operators on H

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