

# ON THE NON-LINEAR COHOMOLOGY OF LIE EQUATIONS. I

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Due to the length of this paper, it is being published in two parts.  
Part II will appear at the beginning of the next issue of this journal.

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## Introduction

The infinitesimal transformations of a Lie pseudogroup, acting on a manifold  $X$ , are solutions of a linear partial differential equation  $R_k$  which is a Lie equation in the tangent bundle  $T$  of  $X$ ; the space  $R_{\infty, x}$  of formal solutions of  $R_k$  at a point  $x \in X$  is a topological Lie algebra and, if the pseudogroup is transitive, it is a transitive Lie algebra in the sense of Guillemin-Sternberg [13].

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<sup>(1)</sup> This work was supported in part by National Science Foundation Grants MPS 72-05055 A 02 and MPS 72-04357.