A GENERALIZATION TO OVERDETERMINED SYSTEMS OF THE NOTION OF DIAGONAL OPERATORS

I. *Elliptic operators*

BY

BARRY MacKICHAN

Massachusetts Institute of Technology, Cambridge, Mass., USA and Duke University, Durham, N.C., USA

Introduction

One of the problems in the theory of overdetermined systems of linear partial differential equations is to prove the existence of local solutions. If $\mathcal{D}$ is a differential operator, we would like to determine when we can solve the inhomogeneous equation $\mathcal{D}u=v$. In general, it is necessary that v satisfy a compatibility condition $\mathcal{D}'v=0$ for some operator $\mathcal{D}'$. We would like to prove that this compatibility condition is not only necessary but also sufficient for the existence of local solutions. That is, if $\underline{E}$, $\underline{F}$, and $\underline{G}$ are the sheaves of germs of differentiable sections of the vector bundles E, F, and G, where $\mathcal{D}$: $\underline{E}\to\underline{F}$ and $\mathcal{D}'$: $\underline{F}\to\underline{G}$, then the complex of sheaves,

$$0 \longrightarrow \theta \longrightarrow \underline{E} \xrightarrow{\mathcal{D}} \underline{F} \xrightarrow{\mathcal{D}'} \underline{G} \tag{1}$$

is exact, where θ is the sheaf of solutions of the homogeneous equation.

D. C. Spencer [7] has shown that, granted certain reasonable assumptions about $\mathcal{D}$, there exists a complex

$$0 \longrightarrow \theta \longrightarrow \underline{C}^0 \xrightarrow{D^0} \underline{C}^1 \xrightarrow{D^1} \dots \xrightarrow{D^{n-1}} \underline{C}^n \longrightarrow 0 \tag{2}$$

of sheaves and of first order differential operators such that the cohomology of (2) at $\underline{C}^1$ is the same as the cohomology of (1) at $\underline{F}$. Thus, it is sufficient to consider the Spencer sequence of $\mathcal{D}$.

In general, the Spencer sequence is not exact, but we would like to show that it is when $\mathcal{D}$ satisfies some other conditions, such as ellipticity. Even in this case, however, it has not been shown that the cohomology of the Spencer sequence is finite dimensional.