# GRAPHS ON UNLABELLED NODES WITH A GIVEN NUMBER OF EDGES

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## 1. Introduction

We write $T_{nq}$ for the number of different graphs on $n$ unlabelled nodes with just $q$ edges. We shall find an asymptotic approximation to $T_{nq}$ for large $n$ and determine the exact range for $q$ for which it holds good. In the graphs we consider, every pair of nodes is joined by just one undirected edge or not so joined, though our method can clearly be extended to other types of graph. If the nodes are labelled, there are $N$ possible edges, where $N = n(n-1)/2$, and the number of graphs with just $q$ edges is

$$F_{nq} = \binom{N}{q} = \frac{N!}{q!\,(N-q)!},$$

the number of ways of selecting $q$ objects out of $N$.

All our statements carry the implied condition "for large enough $n$". The number $q$ is subject to bounds depending on $n$. We use $C$ for a positive number, not always the same at each occurrence, independent of $n$ and $q$. The notations $O(\ )$ and $o(\ )$ refer to the passage of $n$ to infinity and each of the constants implied is a $C$.

We shall prove

THEOREM 1. *The necessary and sufficient condition that*

$$T_{nq} \sim F_{nq}/n! \tag{1.1}$$

*as* $n \to \infty$ *is that*

$$\min(q, N-q)/n - (\log n)/2 \to \infty. \tag{1.2}$$

Pólya [2] proved (1.1) when $|2q-N| = O(n)$, though he appears never to have published his proof. Recently Oberschelp [4] proved (1.1) under the condition that $|2q-N| <$

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