SOME GEOMETRIC AND ANALYTIC PROPERTIES OF HOMOGENEOUS COMPLEX MANIFOLDS

PART I: SHEAVES AND COHOMOLOGY

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This is the first of two papers dealing with homogeneous complex manifolds; since the second work is a continuation of this one, we shall let the following introduction serve for both.

The general problem is to study the geometric, analytic, and function-theoretic properties of homogeneous complex manifolds. The present paper, referred to as Part I, is concerned mainly with sheaves and cohomology; the results here may be viewed as the linear part of the solutions to the questions discussed in the second paper (Part II). In fact, in Part II, using the results of Part I as a guide and first approximation, we utilize a variety of geometric, analytic, and algebraic constructions to treat the various problems which we have posed. A previous paper [11], cited as D. G., was concerned with the differential geometry of our spaces, and the results obtained there will be used from time to time.

The study alone of certain locally free sheaves on these manifolds is a rather interesting one and has been pursued in [4], [5], [16], and [21]. The situation is the following: Let $X = G/U = M/V$ be a homogeneous complex manifold written either as the coset space of complex Lie proups G, U or compact Lie groups M, V where M is semi-simple. Then M acts in any analytic vector bundle $\mathbf{E}^{\varrho}$ (1) associated to the principal fibering $U \to G \to X$ by a holomorphic representation $\varrho: U \to GL(E^{\varrho})$. (Such bundles are called homogeneous vector bundles.) The sheaf cohomology groups $H^*(X, \mathcal{E}^{\varrho})$ are then M-modules by an induced representation ϱ^*; these modules have been determined in [5] and [21] when ϱ is irreducible and X is algebraic, and in other special

(1) The notations used here are explained in § 1.