HERMITIAN VECTOR BUNDLES AND THE EQUIDISTRIBUTION OF THE ZEROES OF THEIR HOLOMORPHIC SECTIONS

BY

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1. Introduction

At present a great deal is known about the value distribution of systems of meromorphic functions on an open Riemann surface. One has the beautiful results of Picard, E. Borel, Nevanlinna, Ahlfors, H. and J. Weyl and many others to point to. (See [1], [2].) The aim of this paper is to make the initial step towards an n-dimensional analogue of this theory.

A natural general setting for the value distribution theory is the following one. We consider a complex n-manifold X and a holomorphic vector bundle E over X whose fiber dimension equals the dimension of X and wish to study the zero-sets of holomorphic sections of E.

When X is compact (and without boundary) then it is well-known that if the zeroes of any continuous section are counted properly then the algebraic sum of these zero-points is independent of the section and is given by the integral of the nth Chern (2) class of E over X: Thus we have

$$\text{Number of zeroes of } s = \int_X c_n(E), \tag{1.1}$$

and this formula is especially meaningful for a holomorphic section because the indexes of all the isolated zeroes of such a section are necessarily positive.

The central question of the value distribution theory is to describe the behavior of the zeroes of holomorphic sections when X is not compact. (For continuous sections there

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(2) With misgivings on the part of the second author we have adopted a terminology now commonly used.