THE COEFFICIENTS OF QUASICONFORMALITY OF DOMAINS IN SPACE

BY

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Dedicated to Professor C. Loewner on his seventieth birthday

1. Introduction

1.1. *Main problem.* Quasiconformal mappings in Euclidean n-space, $n>2$, have been studied rather intensively in recent years by several authors. See, for example, Gehring [4], [5], [6]; Krivov [8]; Loewner [9]; Šabat [14]; Väisälä [17], [18]; and Zorič [20], [21]. It turns out that these mappings have many properties similar to those of plane quasiconformal mappings. On the other hand, there are also striking differences. Probably the most important of these is that there exists no analogue of the Riemann mapping theorem when $n>2$. This fact gives rise to the following two problems. Given a domain D in Euclidean n-space, does there exist a quasiconformal homeomorphism f of D onto the n-dimensional unit ball B^n? Next, if such a homeomorphism f exists, how small can the dilatation of f be?

Complete answers to these questions are known when $n=2$. For a plane domain D can be mapped quasiconformally onto the unit disk B^2 if and only if D is simply connected and has at least two boundary points. The Riemann mapping theorem then shows that if D satisfies these conditions, there exists a conformal homeomorphism f of D onto B^2.

The situation is very much more complicated in higher dimensions, and this paper is devoted to the study of these two questions in the case where $n=3$.

1.2. *Notation.* We let R^3 denote Euclidean 3-space with a fixed orthonormal basis (e_1, e_2, e_3), and we let $\bar{R}^3$ denote the Möbius space obtained by adding the point ∞ to R^3.

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