POINTWISE LIMITS FOR SEQUENCES OF CONVOLUTION OPERATORS

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§ 0. Introduction

(0.1) This paper had its origin in an effort to obtain pointwise inversion formulae for Fourier transforms on a locally compact Abelian group. Does there exist a process for recapturing almost everywhere a function from its Fourier transform? Mean convergence of summability processes for Fourier transforms is of course well known and almost obvious (see for example [12], (20.15)). The whole point of the present paper is to replace mean convergence by pointwise convergence almost everywhere.

In § 1 we present a general theorem on pointwise limits of sublinear operators. Section 2 is concerned with differentiation of indefinite integrals and measures on a class of locally compact groups. In § 3, we obtain single convergence theorems and inversion formulae on the same class of groups. In § 4 we give an analogue of the martingale convergence theorem for singular convolution operators. We combine the foregoing results in § 5 to give *iterated* limit processes for inverting Fourier and Fourier-Stieltjes transforms on an arbitrary locally compact Abelian group or compact group.

(0.2) We follow the notation and terminology of [12] with the following additions. The term "neighbourhood of a point" means "a set whose interior contains that point". Let X be a locally compact Hausdorff space. A *positive Radon measure on* X is a set function ι on all subsets of X as defined in [12], § 11. Measurability of a subset of X for ι is as defined in [12], (11.28). For a measure ϱ that is in $\mathbf{M}(X)$ or is a positive Radon measure on X, and a locally ϱ-integrable function f on X, the symbol $f\varrho$ denotes the measure $A \to \int_A f\,d\varrho$. For a positive real number p and X and ι as just described, $\mathfrak{L}_{p,\,\mathrm{loc}}(X,\iota)$ is the set of all functions f on X such that $f\xi_F \in \mathfrak{L}_p(X,\iota)$ for all compact sets $F \subset X$.

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