# ON THE PERIODICITY THEOREM FOR COMPLEX VECTOR BUNDLES

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## Introduction

The periodicity theorem for the infinite unitary group [3] can be interpreted as a statement about complex vector bundles. As such it describes the relation between vector bundles over $X$ and $X \times S^2$, where $X$ is a compact (1) space and $S^2$ is the 2-sphere. This relation is most succinctly expressed by the formula

$$K(X \times S^2) \cong K(X) \otimes K(S^2),$$

where $K(X)$ is the Grothendieck group (2) of complex vector bundles over $X$. The general theory of these $K$-groups, as developed in [1], has found many applications in topology and related fields. Since the periodicity theorem is the foundation stone of all this theory it seems desirable to have an elementary proof of it, and it is the purpose of this paper to present such a proof.

Our proof will be strictly elementary. To emphasize this fact we have made the paper entirely self-contained, assuming only basic facts from algebra and topology. In particular we do not assume any knowledge of vector bundles or $K$-theory. We hope that, by doing this, we have made the paper intelligible to analysts who may be unacquainted with the theory of vector bundles but may be interested in the applications of $K$-theory to the index problem for elliptic operators [2]. We should point out in fact that our new proof of the periodicity theorem arose out of an attempt to understand the topological significance of elliptic boundary conditions. This aspect of the matter will be taken up in a subsequent paper.(3) In fact for the application to boundary problems we need not only the periodicity theorem but also some more precise results that occur in the course of our present proof.

(1) Compact spaces form the most natural category for our present purposes.

(2) See § 1 for the definition.

(3) See the *Proceedings of the Colloquium on Differential Analysis*, Tata Institute, 1964.