

ON THE MEAN VALUE OF THE ERROR TERM FOR A CLASS OF ARITHMETICAL FUNCTIONS

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§ 1. Introduction

In a recent paper [2] we studied the average order of a large class of arithmetical functions which occur as the coefficients of Dirichlet series which satisfy a functional equation. In this paper we obtain an estimate, in mean, for the error-term associated with such arithmetical functions. Apart from obtaining a number of classical results as special cases, we obtain some new results on certain arithmetical functions in algebraic number theory.

If \mathfrak{L} is an ideal class in an algebraic number field K of degree n , the Dedekind zeta-function of the class \mathfrak{L} is defined by

$$\zeta_K(s, \mathfrak{L}) = \sum_{\mathfrak{a} \in \mathfrak{L}} \frac{1}{(N\mathfrak{a})^s},$$

where the summation is over all non-zero integral ideals in \mathfrak{L} , and if we consider the arithmetical function

$$\sum_{k \leq x} a_k(\mathfrak{L}),$$

where $a_k(\mathfrak{L})$ denotes the number of ideals in \mathfrak{L} of norm k , then it is known, after Weber and Landau [9] that

$$E(x) \equiv \sum_{k \leq x} a_k(\mathfrak{L}) - \lambda x = O(x^{(n-1)/(n+1)}),$$

where λ is the residue of $\zeta_K(s, \mathfrak{L})$ at $s=1$. In this paper we shall show, for example, that if $n=2$, then

$$\frac{1}{x} \int_1^x |E(y)| dy = O(x^{\frac{1}{2}}).$$