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# Pluricomplex Green and Lempert functions for equally weighted poles

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## 1. Introduction

The pluricomplex Green function is an important tool of several variable complex analysis; in particular it provides a fundamental solution for the complex Monge–Ampère equation and information about the complex geometry of domains [8] (see [5] for an exposition of pluricomplex potential theory). For $n \geq 2$, the complex Monge–Ampère equation is non-linear, so studying the several-pole analogue of the Green function (introduced in [6]) is no easy task, see [2], and [1] for some of the few cases where it can be explicitly computed.

Let $\Omega$ be a domain in $\mathbf{C}^n$, and poles and weights be denoted by

$$S = \{(a_1, \nu_1), \dots, (a_N, \nu_N)\} \subset \Omega \times \mathbf{R}_+,$$

where $\mathbf{R}_+ = [0, +\infty)$. The pluricomplex Green function is defined by

$$\begin{aligned} G_S(z) := \sup\{u(z) : u \in \operatorname{PSH}_-(\Omega) \text{ and } \\ u(x) \leq \nu_j \log\|x - a_j\| + C_j, \text{ when } x \to a_j,\ j = 1, \dots, N\}. \end{aligned}$$

Note that if $N=1$ we might as well take $\nu_1 = 1$, in this case $G_S$ is the pluricomplex Green function with one pole.

We also recall the definition of Coman's Lempert function [2]

$$\begin{aligned} l_S(z) := \inf\Bigg\{ \sum_{j=1}^{N} \nu_j \log|\zeta_j| : \varphi(0) = z,\ \varphi(\zeta_j) = a_j,\ j = 1, \dots, N, \\ \text{for some } \varphi \in \mathcal{O}(\mathbf{D}, \Omega) \Bigg\}, \end{aligned}$$

where $\mathbf{D}$ is the unit disc in $\mathbf{C}$.