Function algebras and flows II

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§ 1. When the real line $\mathbf{R}$ acts on a space X there arises a natural notion of analyticity for bounded functions on X. Specifically, we shall say that a bounded function ϕ on X is *analytic* in case the restriction of ϕ to each orbit is a function in $H^\infty(\mathbf{R})$, the space of boundary functions of functions which are bounded and analytic in the upper half plane. Without some global assumptions about the space and the functions, it does not seem possible to say much about the analytic functions. In this paper, which is a sequel to [11], we shall assume that X is a *separable* compact Hausdorff space and that the action of $\mathbf{R}$ on X is continuous. The pair $(X, \mathbf{R})$ will be referred to as a *flow* and for x in X and t in $\mathbf{R}$, the translate of x by t will be denoted by $x+t$. The analytic functions on X considered here are assumed to come from $C(X)$, the space of all continuous complex-valued functions on X, and the algebra which the analytic functions form will be denoted by $\mathfrak{A}$.

Theorem II of [11] asserts that if the flow $(X, \mathbf{R})$ is *strictly ergodic*, meaning that there is a unique probability measure on X which is invariant under the action of $\mathbf{R}$, then $\mathfrak{A}$ is a Dirichlet algebra on X. While the notion of strict ergodicity seems rather special, there is a vague sense in which the strictly ergodic flows are generic among all flows. For example, all minimal almost periodic flows are strictly ergodic; all nil flows are too; and surprisingly it happens that if $\mathbf{R}$ acts measurably on a (standard Borel) measure space Y, if the action preserves a finite measure on Y, and if the action is weakly mixing, then there is a strictly ergodic flow $(X, \mathbf{R})$ which is Borel isomorphic to the action of $\mathbf{R}$ on Y [8]. Our objective in this paper is to identify the maximal ideal space $\mathcal{M}_{\mathfrak{A}}$ of $\mathfrak{A}$ when the flow $(X, \mathbf{R})$ is strictly ergodic. We shall show in Theorem II that if the unique invariant measure is not a point mass then $\mathcal{M}_{\mathfrak{A}}$ is homeomorphic to the quotient space obtained from $X\times[0, 1]$ by identifying the slice $X\times\{0\}$ to a point. This result generalizes the well known theorem of Arens and Singer [1] which describes the maximal ideal spaces for the algebras of analytic almost periodic functions on

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