A multi-dimensional renewal theorem for finite Markov chains

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1. Introduction and results

Let U, L and F be functions from $\mathbf{Z}^d$ into the set of real square matrices of finite dimension N, and let in addition $L(t)$ be positive for each t. Define the convolution $L*U$ by the formula

(1.1) $$L*U(t) = \sum_{t_1+t_2=t} L(t_1)U(t_2),$$

and put

(1.2) $$R = \sum_{n=0}^{\infty} L^{n*},$$

provided the sum converges. Here $L^{0*}=\delta$, where $\delta(0)=1$ (the identity matrix) and $\delta(t)=0$ for $t\neq 0$, and $L^{n*}=L*L^{(n-1)*}$ for $n\geqq 1$.

A solution U of the renewal equation $U-L*U=F$ is then given by $U=R*F$, provided the latter expression converges. The object of the present paper is to study the asymptotic behaviour of $R*F(t)$, as $|t|\to\infty$.

The result can be applied to first passage problems for sums of Markov dependent random variables. See Höglund 1989.

Instead of a function L defined on $\mathbf{Z}^d$ we could equally well have considered a matrix valued measure on $\mathbf{R}^d$, but our restriction will save us some labour because it makes smoothing unnecessary.

The approximation will be expressed in terms of quantities related to the matrices $\Lambda(\theta)$, $\theta\in\Theta$, where

(1.3) $$\Lambda(\theta) = \sum_t e^{\theta\cdot t} L(t)$$

and where Θ denotes the interior of the set of $\theta\in\mathbf{R}^d$ for which this sum converges. Here $\theta\cdot t$ stands for the inner product of θ and t. We shall assume that the function L is *irreducible,* by which we mean that for every i and j in $\{1, \ldots, N\}$ there is a positive integer n and a $t\in\mathbf{Z}^d$ such that $L_{ij}^{n*}(t)>0$. We shall assume that $\Theta\neq\emptyset$