Global parametrices for fundamental solutions of first order pseudo-differential hyperbolic operators

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Dedicated to Lennart Carleson on his sixtieth birthday

Introduction

Global parametrices of fundamental solutions of hyperbolic differential operators were first constructed by Ludwig [7] and for pseudo-differential operators by Duistermaat and Hörmander [2] using Fourier integral operators and canonical relations.

The aim of this paper is to give an elementary construction of global parametrices for fundamental solutions of first order pseudo-differential operators. It is a simplified and corrected version of the construction given in my lectures 1985 at the Nankai university in Tianjin, China (Gårding [3]) and reported on in (Gårding [4]).

Consider a first order pseudo-differential or, more precisely, differential-pseudo-differential hyperbolic operator,

$$Q = D_t + P(t, x, D_x) \tag{1}$$

defined on the product of the real line and a paracompact manifold Ω of dimension n. Here $D_t = \partial_t/i$, $D = \partial_x/i$ with obvious ∂_t and ∂_x and $P(t, x, D_x)$ is a classical first order pseudo-differential operator with real principal symbol $p(t, x, \xi)$ defined on the product of the real line and the cotangent bundle $C = T^*(\Omega)\diagdown 0$ of Ω.

Let Y be a point in Ω with coordinates 0 in a system of coordinates y around Y. A parametrix of a fundamental solution of Q with pole at $(0, Y)$ is a distribution $E(t, x)$ satisfying

$$QE(t, x) \equiv \delta(t)\delta(y)$$

modulo smooth functions. It is sufficient to have $E(t, x) = 0$ when $t < 0$ and let