Poles of $|f(z, w)|^{2s}$ and roots of the B-function

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Introduction

Let $f: (\mathbf{C}^2, \bar{0}) \to (\mathbf{C}, 0)$ be an analytically irreducible germ. To f there is associated its local b-function at $\bar{0}$, denoted $b_f(s)$. Properties of $b_f(s)$ have been found by many authors [1, 2, 7, 11, 12, 14, 15, 16, 17]. In this paper a geometric construction is used to give precise formulae for roots (not *all* roots however) of $b_f(s)$ in terms of the geometry of the branch (i.e. germ of an analytically irreducible plane curve) defined by f at $\bar{0}$.

Brauner showed that the topological properties of a branch are determined by the finite set of integers $(n, \beta_1, \ldots, \beta_g)$ comprising its "characteristic sequence". That is, the topology of the link $\{f=0\} \cap S_\varepsilon$, S_ε a small 3-sphere centered at $\bar{0}$, is completely determined by this sequence. This is discussed in [19, pgs. 5—13]. Moreover, the canonical embedded resolution of f, an important component of the work described here, is also determined by this sequence. This is discussed in [12, Sec. 1].

To state the main result, let $e^{(0)}=n$ and $e^{(i)}=gcd(e^{(i-1)}, \beta_i)$, $i=1, 2, \ldots, g$. For each $i=1, 2, \ldots, g$, define

$$r_i = \frac{\beta_i + n}{e^{(i)}}, \tag{0.1}$$

$$R_i = \frac{\beta_i e^{(i-1)} + \beta_{i-1}(e^{(i-2)} - e^{(i-1)}) + \ldots + \beta_1(e^{(0)} - e^{(1)})}{e^{(i)}}$$

and $\varrho_i = -r_i/R_i$.

In [12, pg. 151], it was shown that if f is the complexification of a real analytic germ at $\bar{0}$ and if $gcd(r_i, R_i)=1$, then ϱ_i was a root of $b_f(s)$. Here, for any germ f as above, and independent of the gcd condition, one shows

Theorem 1. *The ratios* $\varrho_1, \ldots, \varrho_g$ *are roots of* $b_f(s)$.