Bessel potentials and extension of continuous functions on compact sets

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1. Introduction

Let K be a compact subset of R^m. H. Wallin [18] proved that if K has classical α-capacity zero for a certain α, then every $f_0 \in C(K)$ can be extended to a continuous function $f \in W_l^p(R^m)$, where $1 \leqq p < \infty$, and l is a positive integer. The number α depends on m, p and l. He also proved a converse statement. However, his results give a complete solution to this extension problem only when $p=2$ [18, Theorem 3, Theorem 4]. We are going to give a solution to this problem by considering $L_\alpha^p(R^m)$, $1<p<\infty$, $\alpha>0$, α not necessarily an integer. The case studied by H. Wallin is then included since $L_\alpha^p(R^m)=W_\alpha^p(R^m)$, when $1<p<\infty$ and α is a positive integer.

We state our main result in an even more general form by considering potentials relative to general kernels $k(r)$, of L^p-functions. For notations and statement of the theorem, see section 2. See [9] for classical potential theory.

2. Preliminaries and statement of the theorem

We consider R^m with Euclidean norm. All sets are sets of points in R^m. Compact and open sets are denoted by K and V respectively.

The spaces $C(K)$, $C^\infty(V)$, and $C_0^\infty(V)$ are defined in the usual way.

The Lebesgue measure of a set E is denoted by mE and integration with respect to Lebesgue measure is written $\int_E dx$. The spaces $L^p(E)$, $1 \leqq p < \infty$, with norm $\|\cdot\|_{L^p(E)}$ are defined in the usual way. When $E=R^m$ we write L^p and $\|\cdot\|_p$. The class of positive elements in $L^p(E)$ is denoted by $L_+^p(E)$. As a general rule, a sub index $+$ denotes positive elements. The conjugate of p is $q=p/p-1$.

The class A_1 consists of all sets which are measurable for all non-negative

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