An application of a general Tauberian remainder theorem

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1. Introduction

Let Φ be a real-valued, measurable and bounded function on $\mathbf{R}$ and let $F \in L^1(\mathbf{R})$. Introduce the Fouriertransform $\hat{F}$ of F

$$\hat{F}(\xi) = \int_{-\infty}^{\infty} e^{-i\xi x} F(x)\, dx$$

and the convolution

$$\Phi * F(x) = \int_{-\infty}^{\infty} \Phi(x-y) F(y)\, dy.$$

Let us consider a Tauberian relation of the form

$$|\Phi * F(x)| \leqq \varrho(x), \quad x \geqq x_0 \tag{1.1}$$

where $\varrho \searrow$

In an earlier paper [6] a new method was developed and a new set of conditions on $\hat{F}$ were introduced in order to derive an estimate of $|\Phi(x)|$ as $x \to \infty$ from (1.1) and a Tauberian condition for Φ. As an application such results were proved when $1/\hat{F}(\zeta)$, $\zeta = \xi + i\eta$, is analytic in a strip $-\gamma < \eta < \gamma$ around the real axis and the order of magnitude of $1/\hat{F}$ in this strip is known.

In the present paper I use the results in [6] and a lemma for analytic functions proved in Section 2 below to obtain corresponding results when $1/\hat{F}$ is analytic in the strip $0 < \eta < \gamma$ only and the order of magnitude of $1/\hat{F}$ in this strip is known. In this way some new results are obtained. For instance, Theorem 1 in Section 3 below uses no condition on the derivative of $1/\hat{F}$, a condition which is imposed in all earlier theorems of this type (but for the partial result contained in Theorem 1 in [5]). In Theorems 2 and 3 conditions are imposed also on the derivative of $1/\hat{F}$. Theorem 2 extends earlier results of Ganelius and Frennemo and Theorem 3 deals with the case when the 'remainder' $\varrho(x)$ in (1.1) is majorized by $e^{-\alpha x}$ for some $\alpha \geqq \gamma$. In 3.3 I also consider the case when $1/\hat{F}(\zeta)$, $\zeta = \xi + i\eta$, is analytic in a domain $0 < \eta < \gamma(\xi)$ which tapers off at infinity. Theorems 4 and 5 deal with this case.