Hybrids between hyperbolic and elliptic differential operators with constant coefficients

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Introduction

A partial differential operator with constant coefficients is said to be elliptic (hypoelliptic) if it has a fundamental solution which is real analytic (infinitely differentiable) outside the origin. It is said to be hyperbolic if it has a fundamental solution which vanishes outside some proper cone with vertex at the origin. These classes of operators are well known and their properties have been studied in some detail (see e.g. Hörmander [5] and Atiyah—Bott—Gårding [2]). We shall here make a detailed study of a class of operators called hybrid or hyperbolic-elliptic operators, defined by having fundamental solutions that are analytic outside proper cones with vertices at the origin. Like the elliptic and hyperbolic operators these hybrids have principal parts in the same class and weaker lower order terms are the only lower order terms that can be added to a hybrid operator without destroying its character. The hybrids between the hypoelliptic and hyperbolic operators, characterized algebraically by Shirota [9], do not share these simple properties. For them, as for the hypoelliptic operators, the principal part is not a relevant concept.

I shall now list the main results. First some notation and definitions. Points in $\mathbf{R}^n$ will usually be denoted by ξ, η while points in $\mathbf{R}$ will be denoted by s, t. When $P(\xi)$ is a polynomial, $P(D)$, $D=(\partial/i\partial x_1, \ldots, \partial/i\partial x_n)$, denotes the associated differential operator. Homogeneous polynomials will be denoted by a. The class of hybrid or hyperbolic-elliptic operators having fundamental solutions analytic outside proper cones with vertices at the origin, on which $x=0$ or $\langle x, \theta\rangle > 0$ will be denoted by he (θ). The subclass of homogeneous elements in he (θ) is called He (θ). Both these classes can be characterized algebraically. The results, parallell to those of the hyperbolic classes Hyp (θ) and hyp (θ), run as follows: