Jordan decomposition for a class of singular differential operators

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0. Introduction

In 1955 H. L. Turrittin published a theorem on canonical forms of certain differential operators (cf. [1], Theorem I).

We shall not describe that theorem in all detail in this introduction. However, in order to understand the main result of the present paper it is useful to know that Turrittin considers differential operators of the type

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \to \left\{\tau^g \frac{d}{d\tau} + A(\tau)\right\} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

where $g \in N$, $x_i \in \mathbf{C}[[\tau]]$, the ring of formal power series in one variable τ, and A is a square matrix of n rows and columns and elements in $\mathbf{C}[[\tau]]$. Turrittin's statement is roughly as follows: By a convenient "coordinate transformation"

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = P(t) \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix},$$

where y_i and the elements of the matrix $P(t)$ belong to $\mathbf{C}[[t]]$, $t = \tau^{1/p}$ (p positive integer) the differential operator can be expressed as

$$\begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} \to \left\{t^h \frac{d}{dt} + B(t)\right\} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix},$$

where $B(t)$ is a matrix with elements in $\mathbf{C}[[t]]$ having an explicitly prescribed canonical form closely resembling the Jordan canonical form for ordinary linear transformations.