On rearrangements of vector-valued H_1-functions

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1. Introduction[1]

In [4], B. Davis characterized those real functions in $L_1(\mathbf{T})$ which are rearrangements of an H_1-function. Let us identify $\mathbf{T}$ with $\mathbf{R}/\mathbf{Z}$ i.e. with $[0, 1)$ under addition modulo one. Then f can be rearranged to be in $\Re H_1$ if and only if $f_d \in \Re H_1$ where f_d is the decreasing rearrangement of f on $[0, 1)$, and this is if and only if

$$\int_0^1 \frac{|M_1(t)|}{t}\, dt < \infty \tag{1}$$

where

$$M_1(t) = \int_{-t}^{t} f_d(s)\, ds.$$

Here $f_d(s) = f_d(s+1)$, $s<0$. Davis' original proof uses probabilistic methods. Later J. L. Lewis (unpublished) gave an analytic proof. See also [6] for another proof, and also see [5], for related work.

After the initial preparation of the paper, Professor Davis informed the author of the existence of another solution of the rearrangement problem due to O. D. Ceretelli [3], who shows that f has a rearrangement in $\Re H_1$ if and only if

$$\int_1^\infty \frac{|M_2(t)|}{t}\, dt < \infty \tag{2}$$

where

$$M_2(t) = \int_{|f(s)|>t} f(s)\, ds.$$

Ceretelli's results seem to have escaped attention in the West until quite recently; see [7].

In the course of preparing [6] we also found yet another proof which has the virtue of extending naturally to an arbitrary Banach space. In this note we there-

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