Fredholm representations of uniform subgroups

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Introduction

In [8] Mishchenko defined the notion of a Fredholm representation of a discrete group Γ and a map

$$\alpha\colon R(\Gamma) \to K(B\Gamma)$$

from the set of Fredholm representations of Γ to the K-theory of the classifying space $B\Gamma$. For the special case when $B\Gamma$ is homotopy equivalent to a compact manifold of negative curvature, it was proved that the image of α generates $K(B\Gamma)\otimes\mathbf{Q}$. This led to a proof of a conjecture of Novikov concerning rational homotopy invariants.

We give an extension of this result for the case when Γ is a torsionless, uniform subgroup of a non-compact, semisimple Lie group. By enlarging the class $R(\Gamma)$ to include representations which become unitary after projecting to the Calkin algebra, it can be proved that the map is surjective.

1. Fredholm representations

Let H, H_1, H_2 be Hilbert spaces. The symbol $B(H_1, H_2)$ will be used to denote the space of bounded linear operators from H_1 to H_2, and $A(H)$, the Calkin C^*-algebra of H. A representation ϱ of a group Γ on H will be said to become unitary in the Calkin algebra if

$$\varrho(\gamma)^* - \varrho(\gamma^{-1})$$

is compact for all $\gamma\in\Gamma$.

A Fredholm representation of a discrete group Γ is a triple

$$((H_1, \varrho_1), (H_2, \varrho_2), F)$$

where (H_1, ϱ_1) and (H_2, ϱ_2) are representations of Γ on H_1 and H_2, resp., that become unitary in their respective Calkin algebras and $F\colon H_1\to H_2$ is a Fred-