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On the solvability of linear partial differential equations in spaces of hyperfunctions

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It is well known from the theory of linear partial differential equations in spaces of smooth functions and distributions, see Hörmander [11], [12], that the solvability of a differential equation is related to the non-existence of a solution of the homogeneous adjoint equation with compact singular support, and that this may be used to obtain semi-global existence results from the microlocal study of the adjoint operator. In this paper we show that a similar strategy is possible in the framework of hyperfunctions. Actually, we shall consider in this paper the more general case of a system of differential equations without compatibility conditions in the framework of hyperfunctions on a maximally real manifold in $\mathbf{C}^n$ with low regularity.

The first section of the paper may be considered as a continuation of Schapira [26], [27], in which it was shown how functional analysis can be used in the hyperfunction theory of differential operators. We first recall the fact that hyperfunction solvability is insensitive to the geometry of the boundary of the domain (Theorem 1.2) and show that finite dimensional obstruction to solvability never occurs (Theorem 1.3). Then we characterize the hyperfunction solvability of a differential operator in terms of the validity of an *a priori* inequality for the adjoint operator (Theorem 1.4). The main result of this section is perhaps Theorem 1.6 which states that the non-confinement of analytic singularities for the adjoint operator is a sufficient condition for the hyperfunction solvability. This is similar to Theorem 1.2.4 of Hörmander [11].

In Section 2 we give several examples of how the functional analysis statements of Section 1 apply to obtain seemingly new existence theorems or new proofs of classical existence theorems, as corollaries of already available, sometimes deep, microlocal results. Such topics as holonomic systems, hypo-analytic structures or analytic differential equations of principal type on $\mathbf{R}^n$ are touched on. Theorem 2.2

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