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A topological rigidity theorem on open manifolds with nonnegative Ricci curvature

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1. Introduction

Let M be an n-dimensional complete Riemannian manifold with nonnegative Ricci curvature. For a base point $p \in M$ we denote by $B(p,r)$ the open geodesic ball with radius r around p and let $\mathrm{vol}[B(p,r)]$ be its volume. Let ω_n be the volume of the unit ball in the Euclidean space $\mathbf{R}^n$ and define α_M by

$$\alpha_M = \lim_{r \to \infty} \frac{\mathrm{vol}[B(p,r)]}{\omega_n r^n}.$$

It follows from the relative volume comparison theorem [BC], [Gr2] that the limit at the right-hand side in the above equality exists and it does not depend on the choice of p. Thus α_M is a global geometric invariant of M. The manifold M has *large volume growth* if $\alpha_M > 0$. Note that, in this case,

$$\mathrm{vol}[B(p,r)] \geq \alpha_M \omega_n r^n \quad \text{for } p \in M \text{ and } r > 0. \tag{1.1}$$

The structure of complete noncompact Riemannian manifolds with nonnegative Ricci curvature and large volume growth has received much attention. Let M be such an n-dimensional manifold. Li [L] and Anderson [A] have each proven that M has finite fundamental group. Li uses the heat equation while Anderson uses volume comparison arguments to prove this theorem. Perelman [P] has shown that there is a small constant $\varepsilon(n) > 0$ depending only on n such that if $\alpha_M > 1 - \varepsilon(n)$, then M is contractible. It has been shown by Cheeger and Colding [CC] that the condition in Perelman's theorem actually implies that M is diffeomorphic to $\mathbf{R}^n$. Shen [S2] has proven that M has finite topological type, provided that

$$\frac{\mathrm{vol}[B(p,r)]}{\omega_n r^n} = \alpha_M + o\left(\frac{1}{r^{n-1}}\right)$$