A minimum modulus theorem and applications to ultradifferential operators

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In this work we give a minimum modulus theorem which enables us to prove the invertibility of a large class of ultradifferential operators.

It is known that the invertibility of convolution operators defined by ultradistributions S with compact support is equivalent to the existence of a certain lower estimation for the modulus of the Fourier transform of S (see [1], [3], [8], [9]). While usual differential operators with constant coefficients are all invertible even in the space of Schwartz's distributions, the following problem is still open:

Is every ultradifferential operator invertible in the corresponding ultradistributions space or at least in the "union" of all ultradistributions?

In [2] Ch. Ch. Chou positively solved this problem for elliptic ultradifferential operators. For the general case some results are given by the same author in [1]; unfortunately, the invertibility is proved under very restrictive conditions on the considered ultradistributions space.

The aim of this work is to give a general minimum modulus theorem, improving the well-known theorem of L. Ehrenpreis [7] and which yields to an invertibility result in ultradistributions spaces satisfying less restrictive conditions then those of Ch. Ch. Chou. In particular we prove that all ultradifferential operators of class $\{k!\,(\prod_{j=2}^{k} \ln j)^{\alpha}\}$ with $\alpha > 1$, are invertible, while Chou's result works only for $\alpha > 2$.

1. A minimum modulus theorem

Let f be an entire function with $f(0)=1$ and let $a_1, a_2, \dots$ be its zeros indexed such that $|a_1| \leqq |a_2| \leqq \dots$. We denote for each $r > 0$

$$M_f(r) = \sup_{|z|=r} |f(z)|$$

and

$$n_f(r) = \text{the numbers of } a_k \text{ with } |a_k| \leqq r.$$