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Trace expansions for pseudodifferential boundary problems for Dirac-type operators and more general systems

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1. Introduction

One of the purposes of this paper is to prove asymptotic expansions of heat traces

$$\tag{1.1} \begin{gathered} \operatorname{Tr}(\varphi e^{-t\Delta_i}) \sim \sum_{-n\le k<0} a_{i,k} t^{k/2} + \sum_{k=0}^{\infty} (a_{i,k}\log t + a'_{i,k}) t^{k/2} \quad \text{for } t\to 0, \\ \Delta_1 = D_B^* D_B, \quad \Delta_2 = D_B D_B^*, \end{gathered}$$

for general realizations D_B of first-order differential operators D (e.g. Dirac-type operators) on a manifold X with pseudodifferential boundary conditions: $B(u|_{X'})=0$ at the boundary $\partial X = X'$. In (1.1), φ denotes a compactly supported morphism. The coefficients without primes are locally determined, the primed coefficients global.

Such realizations were considered first by Atiyah, Patodi and Singer in [APS] who showed an interesting index formula in the so-called product case, when X is compact. We say that D is *of Dirac-type* when $D=\sigma(\partial_{x_n}+A_1)$ on a collar neighborhood of X', with a unitary morphism σ and a first-order differential operator A_1 such that $A_1 = A + x_n P_1 + P_0$ with A selfadjoint on X' and constant in x_n and the P_j of order j; the *product case* is where $P_1=P_0=0$. The operator B was in [APS] taken equal to the orthogonal projection $\Pi_{\ge}$ onto the eigenspace for A associated with eigenvalues ≥ 0.

For Dirac-type operators on compact manifolds, finite expansions (1.1) (up to $k=0$, with $\varphi=1$ and $a_{i,0}=0$) were shown in [G4], implying the index formula

$$\tag{1.2} \operatorname{index} D_B = a'_{1,0} - a'_{2,0}, \quad \text{when } \varphi = 1 \text{ and } X \text{ is compact.}$$