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Riesz transforms on compact Lie groups, spheres and Gauss space

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Notation. For $x, y \in \mathbf{R}^n$, $x=(x_1, \dots, x_n)$, $y=(y_1, \dots, y_n)$, $|x|=(\sum_{j=1}^n x_j^2)^{1/2}$ is the Euclidean norm of x and $\langle x, y\rangle = \sum_{j=0}^n x_j y_j$ is the inner product of x and y. Sometimes, we write $x \cdot y$ instead of $\langle x, y\rangle$. If $(X, \mathcal{F}, \mu)$ is a measure space, $f: X \to \mathbf{R}^n$ is a measurable function and $p \in [1, \infty)$, the L^p norm of f is defined by $\|f\|_p = \|f\|_{L^p(X, \mathbf{R}^n)} = \left(\int_X |f|^p \, dx\right)^{1/p}$. If S is a linear operator which maps $\mathbf{R}^n$ valued L^p functions on $(X, \mathcal{F}, \mu)$ to $\mathbf{R}^m$ valued L^p functions on $(X_1, \mathcal{F}_1, \mu_1)$, that $\|S\|_p = \sup\{\|Sf\|_p : \|f\|_p = 1\}$ is the operator norm of S. If $X = X_1$ and $\mu = \mu_1$, we denote by $I \oplus S$ the operator with $(I \oplus S)f = (f, Sf)$, the latter being an $\mathbf{R}^{n+m}$ valued function.

Let $\mathcal{A}$ be a linear space of integrable functions on $(X, \mathcal{F}, \mu)$. We denote by $\mathcal{A}_0$ the subspace $\mathcal{A}_0 = \left\{f \in \mathcal{A} : \int_X f \, d\mu = 0\right\}$. If a linear operator S is only defined on $\mathcal{A}_0$, we still denote by $\|S\|_p = \sup\left\{\|Sf\|_p : f \in \mathcal{A}_0,\ \|f\|_p = 1\right\}$. For instance, $C_0^\infty(M) = \left\{f \in C^\infty(M) : \int_M f(x) \, dx = 0\right\}$, if M is a smooth Riemannian manifold and dx denotes the volume element on M. The L^p norm of a measurable vector field U on M is, by definition, the L^p norm of $|U|$, the modulus of U. Unless otherwise specified, $L^p(X)$ and $L_0^p(X)$ will denote spaces of real valued functions on X.

0. Introduction

Let M be a Riemannian manifold without boundary, ∇_M, div_M and $\Delta_M = \operatorname{div}_M \nabla_M$ be, respectively, the gradient, the divergence and the Laplacian associated with M. Then $-\Delta_M$ is a positive operator and the linear operator

$$R^M = \nabla_M \circ (-\Delta_M)^{-1/2} \tag{1}$$

is well defined on $L_0^2(M)$ and, in fact, an isometry in the L^2 norm. If f is a real valued function on M and $x \in M$, then $R^M f(x) \in T_x M$ is a vector tangent to M at x.

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