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Factorization of generalized theta functions in the reducible case

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Introduction

One of the problems in algebraic geometry motivated by conformal field theory is to study the behaviour of moduli space of semistable parabolic bundles on a curve and its generalized theta functions when the curve degenerates to a singular curve. Let X be a smooth projective curve of genus g, and $\mathcal{U}_X$ be the moduli space of semistable parabolic bundles on X, one can define canonically an ample line bundle $\Theta_{\mathcal{U}_X}$ (the theta line bundle) on $\mathcal{U}_X$ and the global sections $H^0(\Theta^k_{\mathcal{U}_X})$ are called generalized theta functions of order k. These definitions can be extended to the case of a singular curve. Thus, when X degenerates to a singular curve X_0, one may ask the question how to determine $H^0(\Theta^k_{\mathcal{U}_{X_0}})$ by generalized theta functions associated to the normalization $\widetilde{X}_0$ of X_0. The so called fusion rules suggest that when X_0 is a nodal curve the space $H^0(\Theta^k_{\mathcal{U}_{X_0}})$ decomposes into a direct sum of spaces of generalized theta functions on moduli spaces of bundles over $\widetilde{X}_0$ with new parabolic structures at the preimages of the nodes. These factorizations and the Verlinde formula were treated by many mathematicians from various points of view. It is obviously beyond my ability to give a complete list of contributions. According to [Be], there are roughly two approaches: infinite and finite. I understand that those using stacks and loop groups are infinite approaches, and working in the category of schemes of finite type is a finite approach. Our approach here should be a finite one.

When X_0 is irreducible with one node, a factorization theorem was proved in [NR] for rank two and generalized to arbitrary rank in [Su]. By this factorization, one can principally reduce the computation of generalized theta functions to the case

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